

### Supplement S6.2 - Noise Margins for the Depletion Load Inverter

Here we calculate the values of  $V_{IH}$ ,  $V_{OL}$ ,  $V_{IL}$  and  $V_{OH}$  for the inverter with a depletion-mode load. Again remember that we are interested in the points in the transfer function at which the slope is equal to -1.

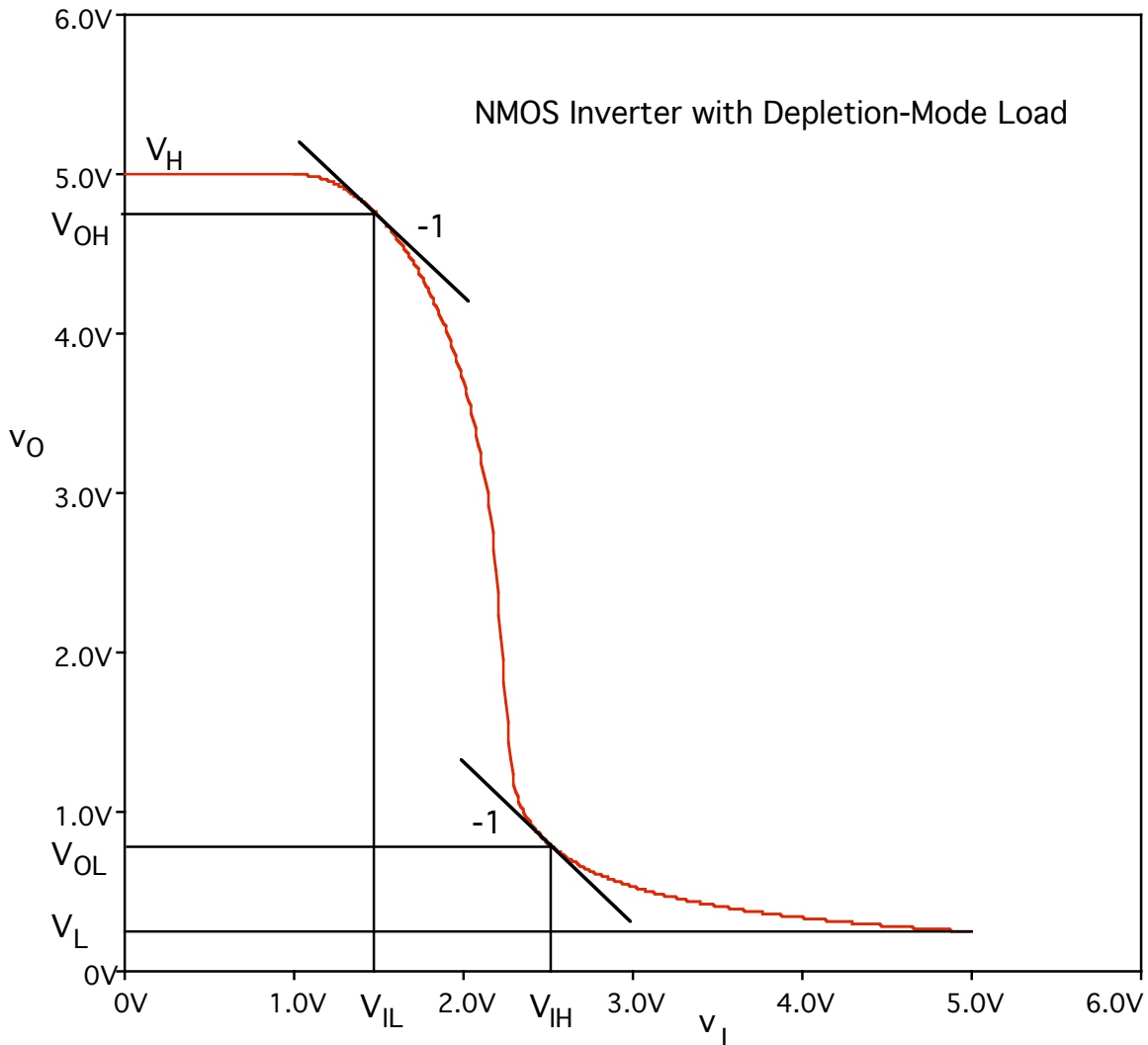


Figure S6.2.1 - PSPICE simulation results for the voltage transfer function of the NMOS depletion load inverter of Fig. 6.29

We will first find  $V_{IL}$  and  $V_{OH}$ . For  $v_I$  near  $V_{IL}$ ,  $v_{DS}$  of  $M_S$  will be large and that of  $M_L$  will be small, so we will assume that the switching device is saturated and the load device is in its linear region. Equating drain currents,  $i_{DS} = i_{DL}$ :

$$\frac{K_S}{2}(v_I - V_{TNS})^2 = K_L \left[ 0 - V_{TNL} - \frac{V_{DD} - v_O}{2} \right] (V_{DD} - v_O) \quad (\text{S6.2.1})$$

In order to simplify the algebra, let us define the ratio  $K_R = \frac{K_S}{K_L}$  representing the relative current drive capability of the switching and load devices in the inverter. Solving for  $v_O$  and taking the derivative yields Eqn. (S6.2.2) in which we have again assumed that  $\frac{\partial V_{TNL}}{\partial v_I} = 0$ .

$$v_O = V_{DD} + V_{TNL} + \sqrt{V_{TNL}^2 - K_R (v_I - V_{TNS})^2} \quad (\text{S6.2.2})$$

$$\frac{\partial v_O}{\partial v_I} = \frac{-K_R (v_I - V_{TNS})}{\sqrt{V_{TNL}^2 - K_R (v_I - V_{TNS})^2}}$$

Setting the derivative equal to -1 at  $v_I = V_{IL}$  yields

$$V_{IL} = V_{TNS} - \frac{V_{TNL}}{\sqrt{K_R^2 + K_R}}$$

$$V_{OH} = V_{DD} + V_{TNL} + \sqrt{V_{TNL}^2 - K_R (V_{IL} - V_{TNS})^2} = V_{DD} + V_{TNL} - \sqrt{\frac{K_R}{K_R + 1}} \quad (\text{S6.2.3})$$

where we have used the negative root. Since we expect that  $V_{OH}$  will approach 5 V, we should improve our estimate of  $V_{TNL}$ :

$$V_{TNL} = 3V + 0.5\sqrt{V} \left( \sqrt{(5.0 + 0.6)V} - \sqrt{0.6V} \right) = 2.20V$$

Using this value of threshold voltage, the values of  $V_{OH}$  and  $V_{IL}$  are 4.78 V and 1.46 V respectively, which agree well with the simulation results in Fig. S6.2.1.

Exercise: What is the value of  $K_R$  for the inverter in Fig. 6.29?

Answer: 4.43

Exercise: Verify the calculations  $V_{IL} = 1.46$  V and  $V_{OH} = 4.78$  V for the inverter in Fig. 6.29. What is the value of  $V_{TNL}$ ?

Answer: -2.22 V

As always, the analysis above was based upon assumptions concerning the operating regions of the load and switching devices, and these assumptions should be checked! The values of  $v_{DS}$  for  $M_S$  and  $M_L$  are 4.78 V and 0.22 V, respectively, which can be used to confirm our operating region assumptions:

$$M_S: V_{GS} - V_{TNS} = 1.46 - 1 = 0.46 \text{ V and } V_{DS} = 0.22 \text{ V} \Rightarrow \text{Linear Region } \checkmark$$

$$M_L: V_{GS} - V_{TNL} = 0 - (-2.20) = 2.20 \text{ V and } V_{DS} = 4.78 \text{ V} \Rightarrow \text{Saturation } \checkmark$$

#### Iterative Update of $V_{IL}$ and $V_{OH}$

It can be seen from Fig. S6.1.1 that the calculated values of  $V_{OH}$  and  $V_{IL}$  are quite close to the SPICE simulation results, but let us try to improve our answers using an iterative numerical update procedure. By examining Eqns. (S6.2.2) and (S6.2.3), we see that one possible iterative sequence is:

1. Choose a starting value for  $V_{TNL}$
2. Calculate the corresponding value of  $V_{IL}$  using (S6.2.3)
3. Use the values  $V_{TNL}$  and  $V_{IL}$  to calculate a new estimate of  $V_{OH}$  using (S6.2.2)
4. Use  $V_{OH}$  to calculate an updated value for  $V_{TNL}$
5. Repeat steps 2 - 3 until the convergence is achieved

Table S6.2.1 - Iterative Update of $V_{IL}$ and $V_{OH}$			
Iteration #	$V_{TNL}$	$V_{IL}$	$V_{OH}$
1	-2.20 V	1.45 V	4.79 V
2	-2.23 V	1.50 V	4.74 V
3	-2.23 V	1.50 V	4.74 V

Starting with  $V_{TNL} = - 2.20 \text{ V}$  and performing two iterations, we obtain the results in Table S6.2.1. The update process converges in three iterations. As was evident

from the simulation results, our initial calculation was very close to the actual answer in Table S6.2.1.

### $V_{IH}$ and $V_{OL}$ for the Depletion Load Inverter

In order to find  $V_{IH}$  and  $V_{OL}$ , the drain currents of the switching and load devices are again required to be equal. For  $v_I = V_{IH}$ , the input will be at a relatively large voltage and the output will be at a relatively small voltage. Thus, we will again assume that  $M_S$  will be in the linear region and expect that  $M_L$  will operate in the saturation region since it should have a large value of  $v_{DS}$ .

$$i_{DS} = i_{DL}$$

$$K_S \left[ v_I - V_{TNS} - \frac{v_O}{2} \right] v_O = \frac{K_L}{2} (V_{TNL})^2 \quad (S6.2.4)$$

We are again interested in the point at which  $dv_I/dv_O = -1$ . Solving for  $v_I$  in terms of  $v_O$  yields:

$$v_I = V_{TNS} + \frac{v_O}{2} + \frac{K_L}{2K_S} \frac{V_{TNL}^2}{v_O} \quad (S6.2.5)$$

$$\frac{\partial v_I}{\partial v_O} = \frac{1}{2} - \frac{1}{2K_R} \frac{V_{TNL}^2}{v_O^2} \quad \text{where} \quad K_R = \frac{K_S}{K_L} \quad (S6.2.6)$$

In this last expression, we have once again neglected the variation of  $V_{TNL}$  with  $v_O$ . Setting the derivative equal to -1 at  $v_I = V_{IH}$  yields

$$V_{OL} = \frac{V_{TNL}}{\sqrt{3K_R}} \quad \text{and} \quad V_{IH} = V_{TNS} + 2V_{OL} = V_{TNS} + \frac{2V_{TNL}}{\sqrt{3K_R}} \quad (S6.2.7)$$

We also know that  $V_{TNL}$  is a function of  $V_{OL}$ :

$$V_{TNL} = V_{T0} + \left[ \sqrt{V_{OL} + 2V_F} - \sqrt{2V_F} \right]. \quad (S6.2.8)$$

$V_{IH}$  can be found by a simple iterative process:

1. Choose an initial value of  $V_{TNL}$
2. Calculate the corresponding values of  $V_{OL}$  and  $V_{IH}$  from (S6.2.7)

3. Use the new value of  $V_{OL}$  and (S6.2.8) to improve the estimate of  $V_{TNL}$
4. Repeat steps 2 and 3 until the process converges

Starting with  $V_{TNL} = -3 \text{ V}$  and performing two iterations, we obtain the results in Table S6.2.1 which also agree very well with those observed in the simulation results in Fig. S6.2.1.

Table S6.2.1 - Iterative Update of $V_{OL}$ and $V_{IH}$				
Iteration #	$V_{OL}$	$V_{TNL}$	$V'_{OL}$	$V_{IH}$
1	---	-3.00 V	0.82 V	2.64 V
2	0.82 V	-2.79 V	0.77 V	2.53 V
3	0.77 V	-2.80 V	0.77 V	2.53 V

Once again, the operating region assumptions should be checked. The values of  $V_{DS}$  for  $M_S$  and  $M_L$  are 0.77 V and 4.23 V, respectively, which can be used to confirm our choice of operating regions:

$$M_S: V_{GS} - V_{TNS} = 2.53 - 1 = 1.53 \text{ V and } V_{DS} = 0.77 \text{ V} \Rightarrow \text{Linear Region} \checkmark$$

$$M_L: V_{GS} - V_{TNL} = 0 - (-2.80) = 2.80 \text{ V and } V_{DS} = 4.23 \text{ V} \Rightarrow \text{Saturation} \checkmark$$

#### Noise Margins

Using the results in Eqns. (S6.2.4) and (S6.2.7), the high state noise margin is expressed by

$$NM_H = V_{OH} - V_{IH} = V_{DD} + V_{TNL}^H \sqrt{\frac{K_R}{K_R + 1}} V_{TNS} + \frac{2V_{TNL}^L}{\sqrt{3K_R}} \quad (S6.2.9)$$

The additional superscripts on  $V_{TNL}$  have been added in Eqn. (S6.2.9) to remind us that two different values of  $V_{TNL}$  are involved in this expression, one for  $V_{OL}$  and one for  $V_{OH}$ !

$$NM_L = V_{IL} - V_{OL} = V_{TNS} - \frac{V_{TNL}^H}{\sqrt{K_R^2 + K_R}} + \frac{2V_{TNL}^L}{\sqrt{3K_R}} \quad (S6.2.10)$$

Here again, the additional superscripts on  $V_{TNL}$  have been added to remind us that two different values of  $V_{TNL}$  are involved in this expression, one for  $V_{OL}$  and one for  $V_{OH}$ ! For the inverter design of Fig. 6.29,  $K_R = (2.06) \times (2.15) = 4.43$ , and using the threshold voltages are from pages 400 and 401 we find:

$$NM_H = 5V + (2.23) - \sqrt{\frac{4.43}{5.43}} + 1 + \frac{2(2.80V)}{\sqrt{3(4.43)}} = 2.25V$$

$$NM_L = 1 - \frac{2.23V}{\sqrt{4.43^2 + 4.43}} + \frac{2.80}{\sqrt{3(4.43)}} = 0.69V$$