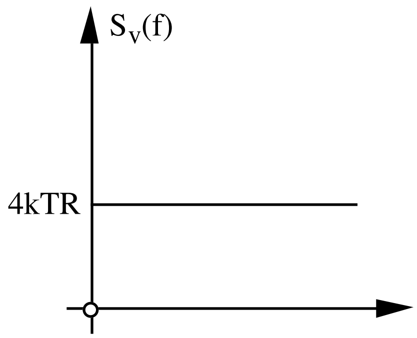
Parseval's Theorem[1]:

The mean square value of a random variable v(t) or i(t) is related to its (one-sided) Power Spectral Density $S_v(f)$ or $S_i(f)$ by

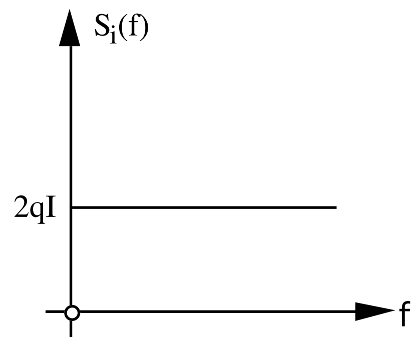
$$\overline{v(t)^2} = \int_0^\infty S_v(f)\, df \quad \text{and} \quad \overline{i(t)^2} = \int_0^\infty S_i(f)\, df$$

White Noise

For "white noise" sources including the thermal noise of resistors or shot noise associated with pn junctions, the power spectra are independent of frequency:



Resistor Thermal Noise
Power Spectrum



Shot Noise
Power Spectrum

The noise power in a frequency band between f_1 and f_2 $\Delta f = f_2 - f_1$ is given by

$$\overline{v(t)^2} = \int_{f_1}^{f_2} 4\text{kTR}\, df = 4kTR\Delta f \quad \text{and} \quad \overline{i(t)^2} = \int_{f_1}^{f_2} 2qI\, df = 2qI\Delta f$$

The bandwidth Δf is also often called B in many publications, $B = f_2 - f_1$:

$$\overline{v(t)^2} = \int_{f_1}^{f_2} 4\text{kTR}\, df = 4kTRB \quad \text{and} \quad \overline{i(t)^2} = \int_{f_1}^{f_2} 2qI\, df = 2qIB$$

Addition of Noise Sources

For $v_s(t) = v_1(t) + v_2(t)$,

$$\overline{v_s^2(t)} = \overline{\left[v_1(t) + v_2(t)\right]^2} = \overline{v_1^2(t)} + \overline{v_2^2(t)} + \overline{2v_1(t)v_2(t)}$$

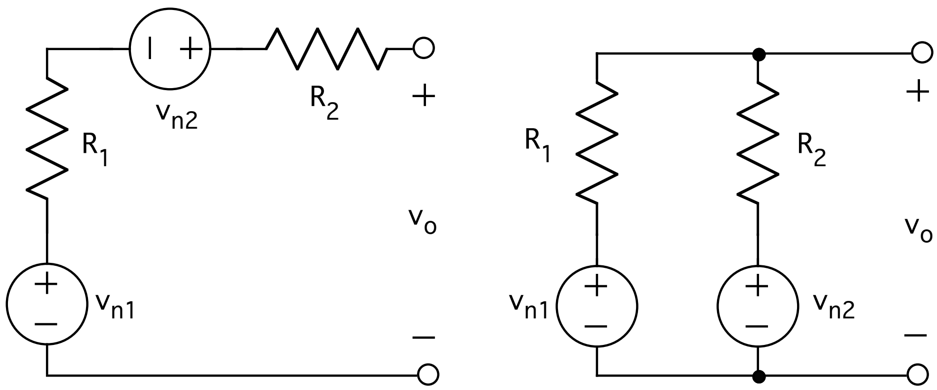
$$\overline{v_s^2(t)} = \overline{v_1^2(t)} + \overline{v_2^2(t)} + 2C_{12}\sqrt{\overline{v_1^2(t)}\,\overline{v_2^2(t)}} \quad \text{with} \quad C_{12} = \frac{\overline{v_1(t)v_2(t)}}{\sqrt{\overline{v_1^2(t)}\,\overline{v_2^2(t)}}} = \text{ correlation coefficient}$$

[1] R. D. Thornton, D. Dewitt, E. R. Chenette and P. E. Gray, *Characteristics and Limitations of Transistors*, SEEC Vol. 4, Wiley: 1966.

C_{12} represents a measure of the correlation between v_1 and v_2, and $-1 \le C_{12} \le +1$. For uncorrelated noise sources, $C_{12} = 0$, and

$$\text{For uncorrelated noise sources:} \quad \overline{v_s^2(t)} = \overline{\left[v_1(t) + v_2(t)\right]^2} = \overline{v_1^2(t)} + \overline{v_2^2(t)}$$

Example of Noise Calculations for Resistor Combinations



Series connection:

$$v_o = v_{n1} + v_{n2} \quad \text{where} \quad \overline{v_{n1}^2} = 4kTR_1B \quad \text{and} \quad \overline{v_{n2}^2} = 4kTR_2B$$

Since the resistors are physically independent,

we expect the individual noise sources to be uncorrelated, and

$$\overline{v_o^2} = \overline{\left(v_{n1} + v_{n2}\right)^2} = \overline{v_{o1}^2} + \overline{v_{o2}^2} = 4kTR_1B + 4kTR_2B = 4kT\left(R_1 + R_2\right)B$$

Parallel connection:

$$v_o = v_{n1}\frac{R_2}{R_1 + R_2} + v_{n2}\frac{R_1}{R_1 + R_2}$$

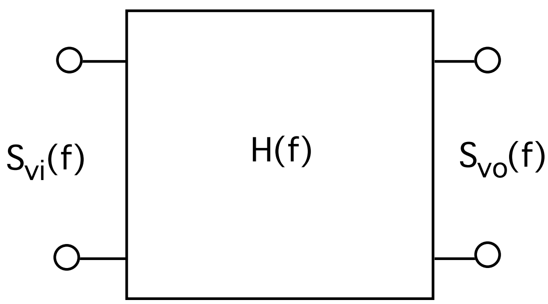
Again, assuming the individual noise sources to be uncorrelated,

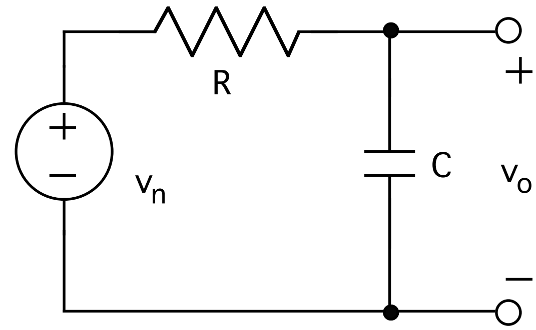
$$\overline{v_o^2} = \overline{\left(v_{n1}\frac{R_2}{R_1 + R_2} + v_{n2}\frac{R_1}{R_1 + R_2}\right)^2} = \overline{v_{o1}^2}\left(\frac{R_2}{R_1 + R_2}\right)^2 + \overline{v_{o2}^2}\left(\frac{R_1}{R_1 + R_2}\right)^2$$

$$\overline{v_o^2} = 4kTB\frac{R_1R_2^2 + R_1^2R_2}{\left(R_1 + R_2\right)^2} = 4kTB\frac{R_1R_2}{R_1 + R_2} = 4kT\left(R_1 \| R_2\right)B$$

Noise Calculations in Frequency Dependent Networks

For a linear frequency dependent network, the output power spectral density $S_{vo}(f)$ is equal to the input power spectrum $S_{vi}(f)$ multiplied by the square of the magnitude or the network transfer function H(f):





$$S_{vo}(f) = S_{vi}(f)\,\left|H(f)\right|^2$$

Consider the low-pass filter in the figure above

$$S_{vi} = 4kTR \qquad H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC} \qquad \left|H(f)\right|^2 = \frac{1}{1+(2\pi RCf)^2}$$

$$S_{vo}(f) = \frac{4kTR}{1+(2\pi RCf)^2} \qquad \overline{v_o^2} = \int_0^\infty \frac{4kTR}{1+(2\pi RCf)^2}\,df = \frac{kT}{C}$$

Effective Noise Bandwidth B

For a given noise power, the effective noise bandwidth is defined as $\text{ENB} = \dfrac{\overline{v_o^2}}{4kTR}$

For the single-pole low-pass filter above with cutoff frequency $\omega_o = \dfrac{1}{RC}$,

$$\text{ENB} = \frac{kT/C}{4kTR} = \frac{1}{4RC} = \frac{\pi}{2} f_o$$

Available Noise Power

The maximum power available from a resistor is $\overline{v_n^2} = kTB$

In a 1-Hz bandwidth at room temperature, the available power corresponds to

$$\overline{v_n^2} = \left(1.38x10^{-23}\,J/K\right)\left(300K\right)\left(1Hz\right) = 4.14x10^{-21}W$$

This value is often expressed in dBm as: $10\log\left[\dfrac{4.14x10^{-21}W}{0.001W}\right] = -174dBm$