Supplement S6.1 - Noise Margins for the Saturated Load Inverter

We will now calculate the values of the V_{IL}, V_{OH}, V_{IH} and V_{OL} for the inverter with a saturated load device. Remember that V_{IH} and V_{IL} are defined by the points in the transfer function at which the slope is equal to -1. In Fig. S6.1.1 the slope of the transfer function abruptly changes as M_S begins to conduct at the point where $v_I = V_{TNS}$. This point defines V_{IL}:

$$V_{IL} = V_{TNS} = 1\text{ V} \quad \text{for} \quad V_{OH} = V_H = V_{DD} - V_{TNL} = 3.39\text{ V}.$$

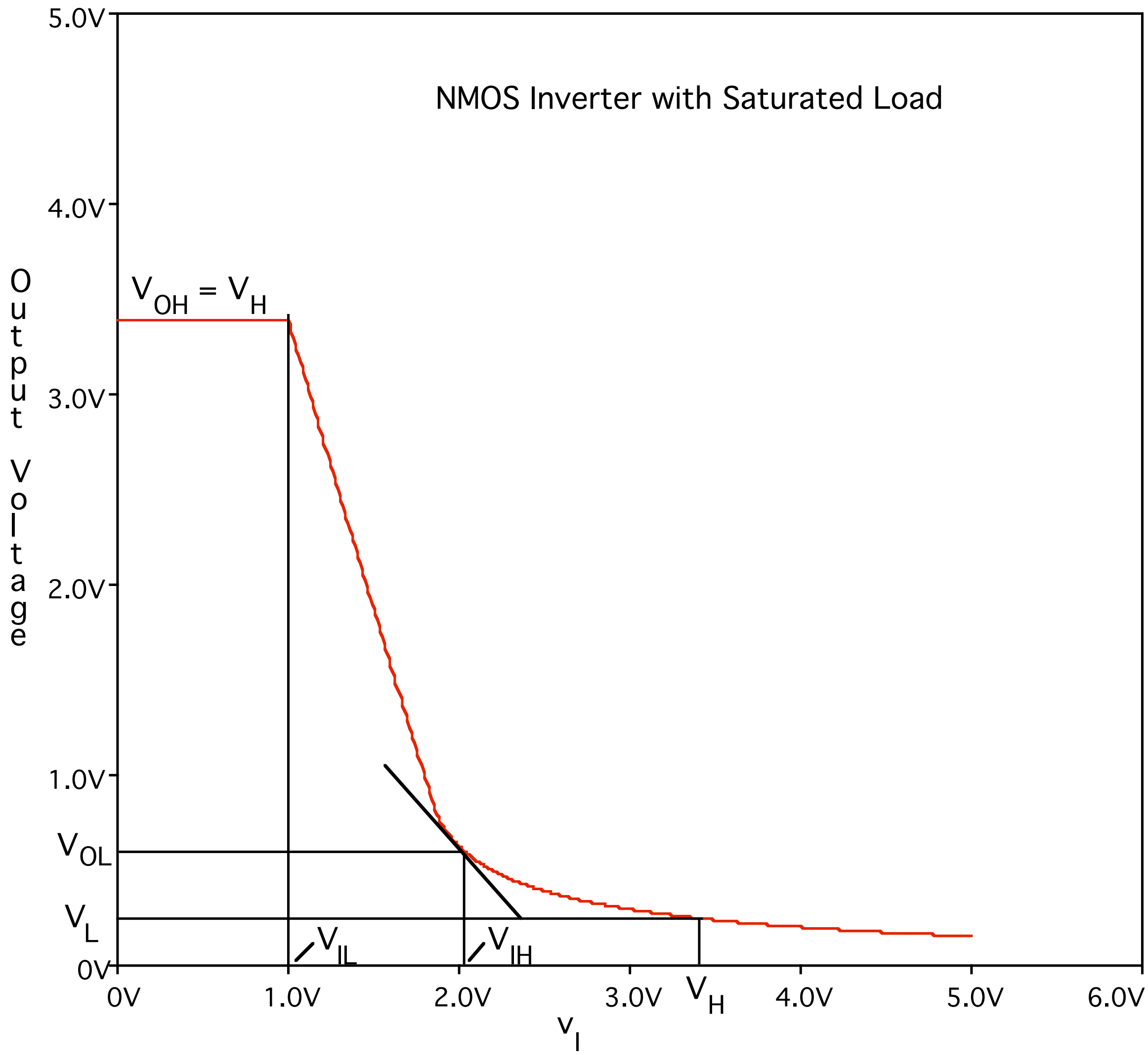

Figure S6.1.1 - PSPICE simulation of the voltage transfer function for the NMOS inverter with saturated load

Next let us find V_{IH}. In order to find a relationship between v_I and v_O, we observe that the drain currents in the switching and load devices must be equal. At $v_I = V_{IH}$, the input will be at a relatively high voltage, and the output will be at a relatively low voltage. Thus, we can guess that M_S will be in the linear region, and we already know that the circuit connection forces M_L to operate in the saturation region. Equating drain currents in the switching and load transistors:

$$i_{DS} = i_{DL}$$

$$K_S\left(v_I - V_{TNS} - \frac{v_O}{2}\right)v_O = \frac{K_L}{2}\left(V_{DD} - v_O - V_{TNL}\right)^2$$

with (S6.1.1)

$$K_S = K_n^{'}\left(\frac{W}{L}\right)_S \quad and \quad K_L = K_n^{'}\left(\frac{W}{L}\right)_L$$

The point of interest is $\frac{\partial v_O}{\partial v_I} = -1$, but solving for the value of v_O can be quite tedious.

Since the derivatives are smooth, continuous and non-zero, we will assume that $\frac{\partial v_O}{\partial v_I} = \left(\frac{\partial v_I}{\partial v_O}\right)^{-1}$, and will solve for v_I in terms of v_O:

$$v_I = V_{TNS} + \frac{v_O}{2} + \frac{1}{2K_R v_O}\left(V_{DD} - v_O - V_{TNL}\right)^2 \quad \text{where} \quad K_R = \frac{K_S}{K_L}$$

or

$$v_I = V_{TNS} + \frac{v_O}{2} + \frac{1}{2K_R}\left[\frac{\left(V_{DD} - V_{TNL}\right)^2}{v_O} - 2\left(V_{DD} - V_{TNL}\right) + v_O\right]$$

and (S6.1.2)

$$\frac{\partial v_I}{\partial v_O} \cong \frac{1}{2} + \frac{1}{2K_R}\left[-\frac{\left(V_{DD} - V_{TNL}\right)^2}{v_O^2} + 1\right]$$

In this last expression, the dependence of V_{TNL} on v_O has been neglected for simplicity. (This approximation will be justified shortly.)

Setting the derivative equal to -1 at $v_I = V_{IH}$ yields

$$-1 = \frac{1}{2} + \frac{1}{2K_R}\left[-\frac{(V_{DD} - V_{TNL})^2}{v_O^2} + 1\right]$$

and solving for $V_{OL} = v_O$ for $v_I = V_{IH}$ yields:

$$V_{OL} = \frac{V_{DD} - V_{TNL}}{\sqrt{1 + 3K_R}} \quad \text{and} \quad V_{IH} = V_{TNS} + \frac{V_{OL}}{2} + \frac{(V_{DD} - V_{OL} - V_{TNL})^2}{2K_R V_{OL}} \tag{S6.1.3}$$

For the inverter design of Fig. 6.26 with V_{TNL} = 1 V, we find

$$V_{OL} = \frac{V_{DD} - V_{TNL}}{\sqrt{1 + 3K_R}} = \frac{V_{DD} - V_{TNL}}{\sqrt{1 + 3\frac{(W/L)_S}{(W/L)_L}}} = \frac{(5 - 1)V}{\sqrt{1 + 3(3.53)(3.39)}} = 0.66V$$

$$V_{IH} = 1 + \frac{0.66V}{2} + \frac{1}{2(3.53)(3.39)}\frac{1}{0.66V}(5 - 0.66 - 1)^2 V^2 = 2.04V$$

With these values we can check our assumption of the operating region of M_S:

$$V_{GS} - V_{TNS} = 2.04 - 1 = 1.04 \text{ V} \quad \text{and} \quad V_{DS} = 0.66 \text{ V}.$$

Since $V_{DS} < V_{GS} - V_{TN}$, the linear region assumption was correct.

In Fig. S6.1.1, it can be seen that the calculated values of V_{IH} and V_{IL} agree well with SPICE simulation results. Thus, the approximation of neglecting the voltage dependence of the threshold voltage of the load device does not introduce significant error.

However, if we so desire, we can use an iterative procedure to improve on the estimates above. The calculated value of V_{OL} can be used to determine a better estimate for V_{TNL}, and this new value of V_{TNL} can then be used to improve the estimate of V_{IH}. For V_{OL} = 0.66V,

$$V_{TNL} = 1V + 0.5\sqrt{V}\left(\sqrt{(0.66 + 0.6)V} - \sqrt{0.6V}\right) = 1.17 \ V$$

and the new values of V_{OL} and V_{IH} from Eqn. (S6.1.3) are equal to

$$V_{OL} = 0.63 \text{ V} \quad \text{and} \quad V_{IH} = 1.99 \text{ V}.$$

This process can be continued, but it has already converged as indicated in Table 7.2 below. (Note that this procedure is another example of an algorithm that is very easy to implement using MATLAB or a spreadsheet.)

Table S6.1.1 - Iterative Update of V_{OL} and V_{IH}

Iteration #	V_{OL}	V_{TNL}	V'_{OL}	V_{IH}
0	---	1.00 V	0.66 V	2.04 V
1	0.66 V	1.17 V	0.63 V	1.99 V
2	0.63 V	1.17 V	0.63 V	1.99 V

$$NM_L = V_{IL} - V_{OL} = 1 - 0.63 = 0.37 \text{ V}$$

$$NM_H = V_{OH} - V_{IH} = 3.39 – 1.99 = 1.40 \text{ V}$$